

# New experiments demand for a more precise analysis of continuum spectrum in ${}^6\text{He}$ : technical details and formalism

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A simple three-body model of  ${}^6\text{He}$  is extended to include  $sd$ -continuum states in the picture in addition to the already investigated  $p$ -states. The role of different continuum components in the weakly bound nucleus  ${}^6\text{He}$  is studied by coupling unbound  $spd$ -waves of  ${}^5\text{He}$  by using simple pairing contact-delta interaction. The main focus of this paper is to outline the procedure that allows the calculation of different configurations of  ${}^6\text{He}$  including continuum states and to set up basic ingredients for computations. The method and results discussed here will be used for the calculation of monopole, dipole, quadrupole and octupole response of  ${}^6\text{He}$ .

PACS numbers: 21.10.Gv, 21.10.Ky, 26.60.Cs

## I. INTRODUCTION

Motivated by the recent experimental measurements at GANIL [1] and in other laboratories [2] on continuum resonances in  ${}^6\text{He}$ , we have developed a simple model [3] to study the weakly bound ground state and low lying continuum states of  ${}^6\text{He}$  by coupling two unbound  $p$ -waves of  ${}^5\text{He}$ . Recently we have extended the model space with inclusion of  $sd$ -continuum waves of  ${}^5\text{He}$  [5, 6]. The large basis set of these  $spd$ -continuum wavefunctions are used to construct the two-particle  ${}^6\text{He}$  ground state  $0^+$  emerging from five different possible configurations i.e.  $(s_{1/2})^2$ ,  $(p_{1/2})^2$ ,  $(p_{3/2})^2$ ,  $(d_{3/2})^2$  and  $(d_{5/2})^2$ . The simple pairing contact-delta interaction is used and pairing strength is adjusted to reproduce the bound ground state of  ${}^6\text{He}$ . Preliminary results shows how the ground state displays collective nature by taking contribution from five different oscillating continuum states that sum up to give an exponentially decaying bound wavefunction [5]. We have also studied several properties of this state: the mean square distance between the valence nucleons and the mean square distance of their centre of mass w.r.t core [5]. In the present paper, rather than concentrating on the physics or on the results (that will be presented somewhere else), we will describe the computational procedure in detail and we will discuss some coefficients that are needed in the computations.

## II. PROCEDURE

The procedure adopted for these calculations is explained in Fig. (1) with the help of a flow chart diagram. It is divided in blocks that correspond to the various codes used. In each block yellow background indicates input lines where data must be passed to the code. Several types of inputs are required: most of them are integers or real number (mostly in convenient nuclear units) that must be set case-by-case, others are strings of text. Most input data are read in the input file.

### A. Block 1

Block 1 calculates the  $spd$ -continuum single-particle states ( $E_C > 0$ ) of  ${}^5\text{He}$  with Woods-Saxon potential + spin-orbit potential [5], with energies from 0.1 to 10.0 MeV on a radial grid that goes from 0.1 to 100.0 fm (notice that this amount to 2.2 Mb of data for each component). Examples of these wave functions are shown in Fig. (2), where the  $s_{1/2}$ ,  $d_{5/2}$  and  $d_{3/2}$  oscillating continuum waves are displayed as a function of  $r$  in the range of 0 – 40 fm for continuum energies 1, 3, 5, 7, 9 and 10 MeV.

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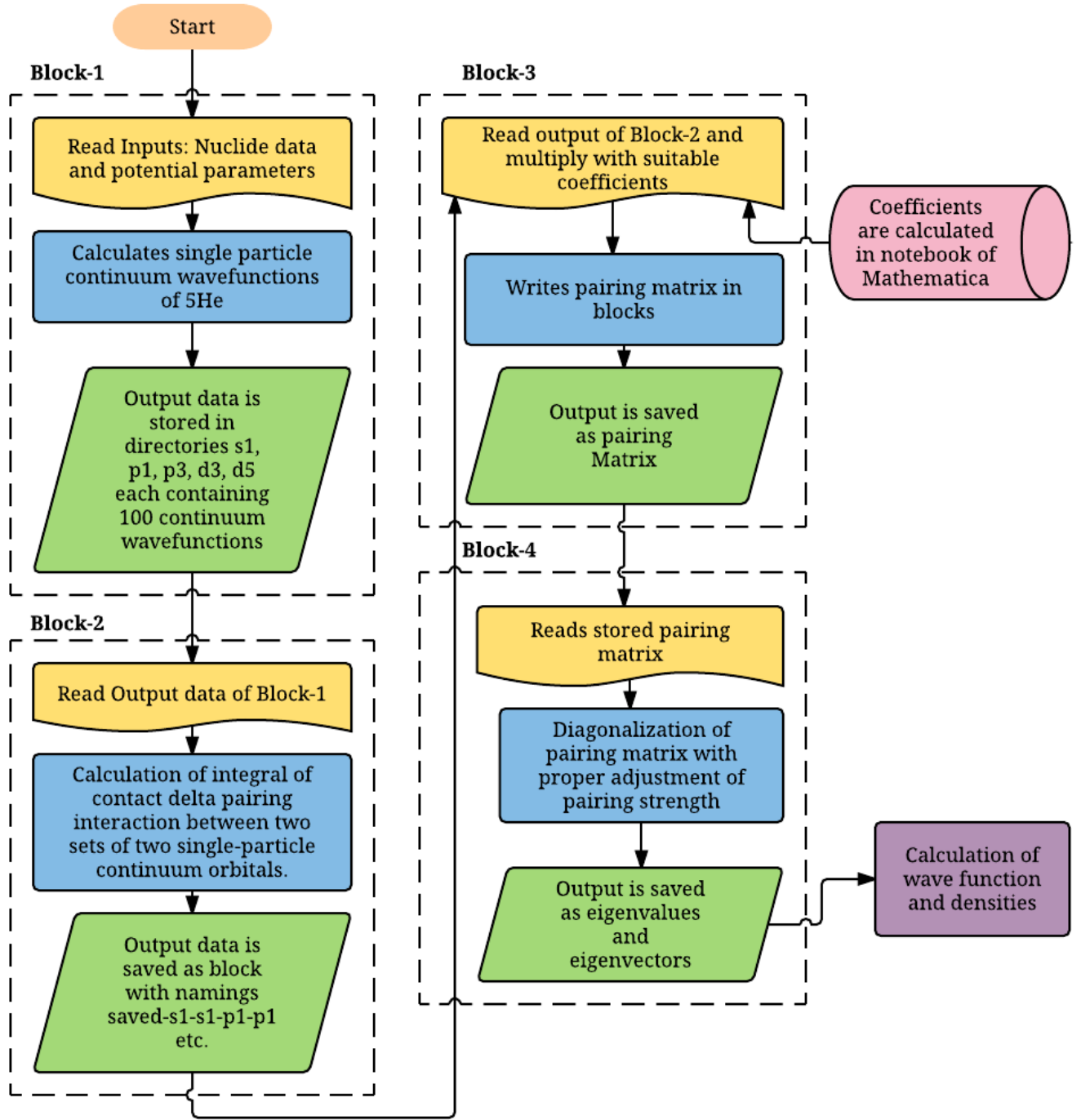


FIG. 1: (Color online) Flow chart diagram of procedure followed with series of codes used. Blocks are indicated with dashed black boxes. Inside each block data reading is indicated as light yellow cards, algorithms are indicated as blue rectangles and outputs in green.

## B. Block 2

By using the midpoint method as a discretization recipe, the wave functions are normalized to a Dirac delta in energy with an energy spacing of 2.0, 1.0, 0.5, 0.2, and 0.1 MeV corresponding to block basis dimensions of  $N = 5, 10, 20, 50$ , and 100 respectively. Two-particle states are constructed with proper couplings to  $J = 0^+, 1^-, 2^+, 3^-$ . Then integral of contact delta pairing interaction between two sets of two single-particle continuum orbitals are calculated. The

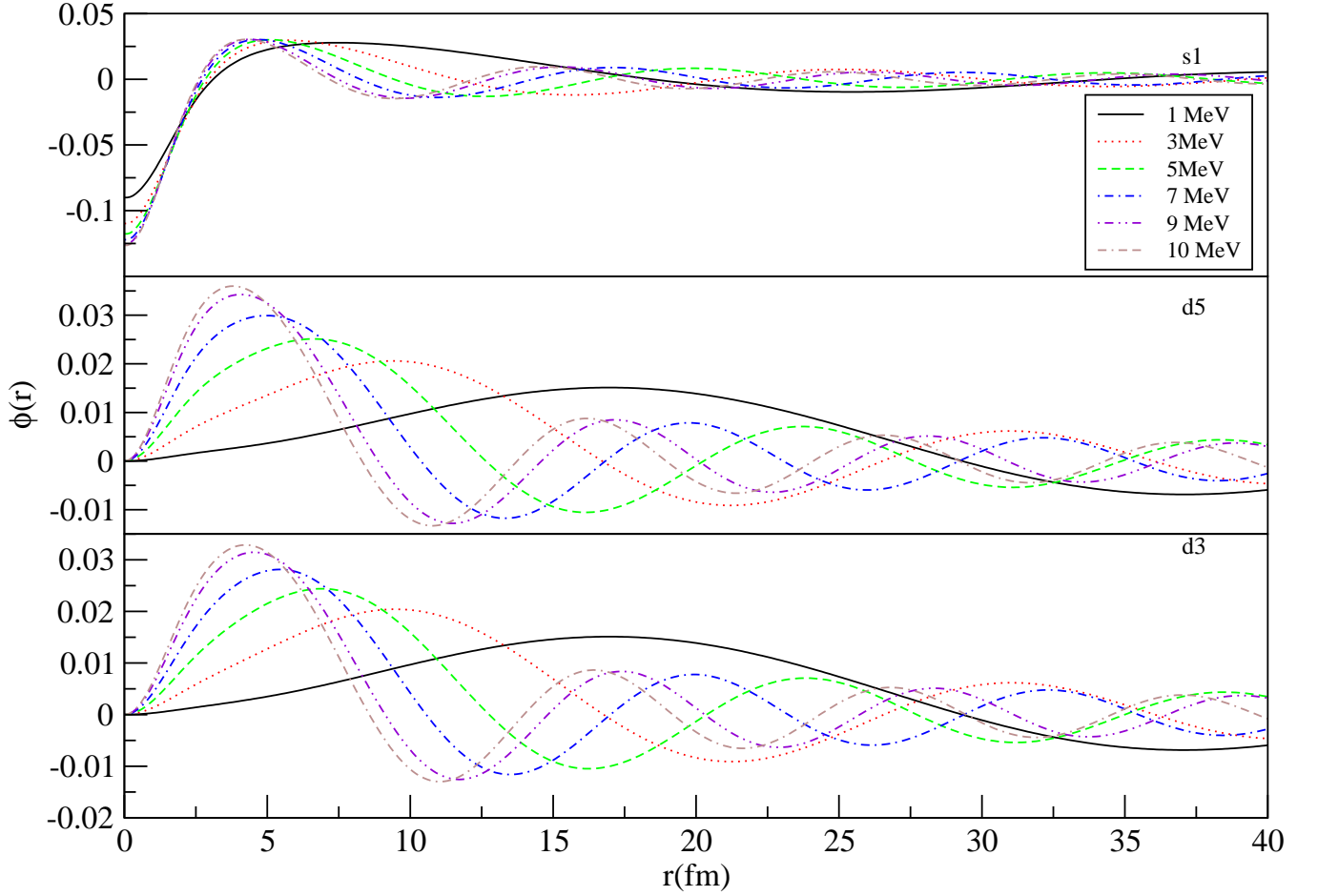


FIG. 2: (Color online)  ${}^5\text{He}$  sd-continuum waves as a function of radial variable for continuum energies 1, 3, 5, 7, 9 and 10 MeV. Each panel is labeled by  $l$  and  $2j$ .

output is saved in matrix blocks. For ground state  $J=0^+$  and  $J=1^-$ , it amounts to calculation of 15 matrix blocks (approximately  $\sim 9$  Gb of data), for  $J=2^+$  state it amounts to calculation of 28 matrix blocks, which is a hard computational task and for  $J=3^-$ , it amounts to calculation of 6 matrix blocks.

### C. Block 3

It simply reads the blocks evaluated in previous block and multiply them with appropriate coefficients which are evaluated in separate mathematica notebook and calculates the matrix elements of pairing matrix. The output pairing matrix is huge data set file ( $\sim 21$  Gb of data for the largest case in ground state).

### D. Block 4

It simply diagonalizes the pairing matrix with standard routines to give eigenvalues and eigenvectors. The coefficient of the  $\delta$ -contact matrix,  $G$ , has been adjusted to reproduce the correct ground state energy each time. The actual pairing interaction  $g$  is obtained by correcting with a factor that depends on the aforementioned spacing between energy states [6]. The biggest adopted basis size gives a fairly dense continuum in the region of interest. The output of block 4 is further used for calculation of ground state wavefunction, transition probabilities and two particle densities.

### III. ${}^6\text{He}$ WAVEFUNCTIONS AND MATRIX ELEMENTS OF PAIRING INTERACTION

The simple model [3] with two non interacting particles in the above single particle levels of  ${}^5\text{He}$  produces different parity states (see Table 1 of [5]). The two-particle wave functions are constructed by tensor coupling of two continuum states of  ${}^5\text{He}$ . The five states of  ${}^5\text{He}$  are not discrete, but rather depend on the energies of the continuum orbitals. Each single particle continuum wavefunction is given by

$$\phi_{\ell,j,m}(\vec{r}, E_C) = \phi_{\ell,j}(r, E_C) [Y_{\ell m}(\Omega) \times \chi_{1/2, m_s}]_m^{(j)} \quad (1)$$

The combined tensor product of these two is given by

$$\psi_{JM}(\vec{r}_1, \vec{r}_2) = [\phi_{\ell_1, j_1, m_1}(\vec{r}_1, E_{C1}) \times \phi_{\ell_2, j_2, m_2}(\vec{r}_2, E_{C2})]_M^{(J)} \quad (2)$$

In  $LS$ -coupling for  $\ell_1 \neq \ell_2$  the antisymmetric wavefunction  $\psi(\ell_1 \ell_2 SLJM)$  is given by

$$\psi(\ell_1 \ell_2 SLJM) = \frac{1}{\sqrt{2}} \sum_{M_S, M_L} \langle SM_S LM_L | SLJM \rangle \times [\phi_{12}(\ell_1 \ell_2 LM_L) \chi_{12}(s_1 s_2 SM_S) - \phi_{21}(\ell_2 \ell_1 LM_L) \chi_{21}(s_2 s_1 SM_S)] \quad (3)$$

The matrix elements due to mutual interaction  $V_{12}$  in  $LS$ -coupling of two particles are given by

$$\begin{aligned} \langle \ell_a \ell_b SLJM | V_{12} | \ell_c \ell_d S' L' J' M' \rangle &= \sum \langle SM_S LM_L | SLJM \rangle \langle S' M'_S L' M'_L | S' L' J' M' \rangle \\ &\langle s_1 m_{s_1} s_2 m_{s_2} | s_1 s_2 SM_S \rangle \langle s'_1 m'_{s_1} s'_2 m'_{s_2} | s'_1 s'_2 S' M'_S \rangle \langle \ell_a m_a \ell_b m_b | \ell_a \ell_b LM_L \rangle \langle \ell_c m_c \ell_d m_d | \ell_c \ell_d L' M'_L \rangle \\ &\sum_{\ell_m} (-1)^{2(\ell-m)} \begin{pmatrix} \ell & \ell_a & \ell_b \\ -m & m_a & m_b \end{pmatrix} \begin{pmatrix} \ell & \ell_c & \ell_d \\ -m & m_c & m_d \end{pmatrix} \langle \ell \| Y_{\ell_a} \| \ell_b \rangle^* \langle \ell' \| Y_{\ell_c} \| \ell_d \rangle \int R_{n_a \ell_a}^*(r) R_{n_b \ell_b}^*(r) \frac{1}{r^2} R_{n_c \ell_c}(r) R_{n_d \ell_d}(r) dr \end{aligned} \quad (4)$$

An attractive pairing contact delta interaction has been used,  $V_{12} = -g\delta(\vec{r}_1 - \vec{r}_2)$  for simplicity, because we can reach the goal with only one parameter adjustment.

### IV. RESULTS

The major ingredients for the complete study of  ${}^6\text{He}$  are the matrix elements of pairing interaction. These correspond to the radial integrals and to the coefficients (calculated in Mathematica notebook mentioned in block 3 of procedure). For ground state these coefficients are used for calculating the contribution of various configurations and some ground state properties (see Table 2 and Table 3 of [5]). The coefficients of the matrix elements of Eq. (4) for  $0^+, 1^-, 2^+$  and  $3^-$  are summarized in Tables 1 to 4 below. These coefficients will lead to the calculation of higher excited states in low lying continuum of  ${}^6\text{He}$ . For all these states the coefficients of matrix elements are calculated for upper diagonal part of the matrix only due to symmetry.

TABLE I: Coefficients of ground and continuum ( $0^+$ ) states of  ${}^6\text{He}$ .

$s_1 s_1 - s_1 s_1$ $1/2\pi$	$s_1 s_1 - p_1 p_1$ $-1/2\pi$	$s_1 s_1 - p_3 p_3$ $-1/\sqrt{2}\pi$	$s_1 s_1 - d_3 d_3$ $1/\sqrt{2}\pi$	$s_1 s_1 - d_5 d_5$ $\sqrt{3}/2\pi$
	$p_1 p_1 - p_1 p_1$ $1/2\pi$	$p_1 p_1 - p_3 p_3$ $1/\sqrt{2}\pi$	$p_1 p_1 - d_3 d_3$ $-1/\sqrt{2}\pi$	$p_1 p_1 - d_5 d_5$ $-\sqrt{3}/2\pi$
		$p_3 p_3 - p_3 p_3$ $1/\pi$	$p_3 p_3 - d_3 d_3$ $-1/\pi$	$p_3 p_3 - d_5 d_5$ $-\sqrt{3}/2\pi$
			$d_3 d_3 - d_3 d_3$ $1/\pi$	$d_3 d_3 - d_5 d_5$ $\sqrt{3}/2\pi$
				$d_5 d_5 - d_5 d_5$ $3/2\pi$

TABLE II: Coefficients of  $1^-$  states of  ${}^6\text{He}$ .

$s_1p_1 - s_1p_1$ $1/6\pi$	$s_1p_1 - s_1p_3$ $1/3\sqrt{2}\pi$	$s_1p_1 - p_1d_3$ $-1/3\sqrt{2}\pi$	$s_1p_1 - p_3d_3$ $2/3\sqrt{10}\pi$	$s_1p_1 - p_3d_5$ $-1/\sqrt{10}\pi$
	$s_1p_3 - s_1p_3$ $1/3\pi$	$s_1p_3 - p_1d_3$ $-1/3\pi$	$s_1p_3 - p_3d_3$ $1/3\sqrt{5}\pi$	$s_1p_3 - p_3d_5$ $-1/\sqrt{5}\pi$
		$p_1d_3 - p_1d_3$ $1/3\pi$	$p_1d_3 - p_3d_3$ $-1/3\sqrt{5}\pi$	$p_1d_3 - p_3d_5$ $1/\sqrt{5}\pi$
			$p_3d_3 - p_3d_3$ $1/15\pi$	$p_3d_3 - p_3d_5$ $-1/5\pi$
				$p_3d_5 - p_3d_5$ $3/2\pi$

TABLE III: Coefficients of  $3^-$  states of  ${}^6\text{He}$ .

$p_1d_5 - p_1d_5$ $3/14\pi$	$p_1d_5 - p_3d_3$ $-3\sqrt{3/10}/7\pi$	$p_1d_5 - p_3d_5$ $3/7\sqrt{5}\pi$
	$p_3d_3 - p_3d_3$ $9/35\pi$	$p_3d_3 - p_3d_5$ $-3\sqrt{6}/35\pi$
		$p_3d_5 - p_3d_5$ $6/35\pi$

## V. CONCLUSION

We have outlined the method and the formulation that we used to calculate  ${}^6\text{He}$  states based on a continuum basis of  ${}^5\text{He}$ . We have discussed the computational procedure in some detail and we have tabulated the coefficients that are needed in the calculation of matrix elements of the pairing interaction. We intend to use these coefficients to study the electromagnetic response of  ${}^6\text{He}$  for transitions to the continuum (monopole, dipole, quadrupole etc. strength distributions), where the separate contribution of different configurations will be evaluated. This will allow us to make predictions on the continuum spectrum of this nucleus.

## VI. ACKNOWLEDGEMENTS

We would like to thank A.Vitturi, R.Chatterjee, J.A.Lay and Sukhjeet Singh for useful suggestions. J.Singh gratefully acknowledges the financial support from Fondazione Cassa di Risparmio di Padova e Rovigo (CARIPARO).

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TABLE IV: Coefficients of  $2^+$  states of  ${}^6\text{He}$ .

$s_1d_3 - s_1d_3$ $1/6\pi$	$s_1d_3 - s_1d_5$ $-\sqrt{3/2}/5\pi$	$s_1d_3 - p_1p_3$ $-1/5\pi$	$s_1d_3 - p_3p_3$ $-1/5\pi$	$s_1d_3 - d_3d_3$ $1/5\pi$	$s_1 - d_3d_3d_5$ $\sqrt{3/7}/5\pi$	$s_1d_3 - d_5d_5$ $2\sqrt{3/7}/5\pi$
	$s_1d_5 - s_1d_5$ $3/10\pi$	$s_1d_5 - p_1p_3$ $\sqrt{3/2}/5\pi$	$s_1d_5 - p_3p_3$ $\sqrt{3/2}/5\pi$	$s_1d_5 - d_3d_3$ $-\sqrt{3/2}/5\pi$	$s_1d_5 - d_3d_5$ $-3/5\sqrt{14}\pi$	$s_1d_5 - d_5d_5$ $-3\sqrt{2/7}/5\pi$
		$p_1p_3 - p_1p_3$ $1/5\pi$	$p_1p_3 - p_3p_3$ $1/5\pi$	$p_1p_3 - d_3d_3$ $-1/5\pi$	$p_1p_3 - d_3d_5$ $-\sqrt{3/7}/5\pi$	$p_1p_3 - d_5d_5$ $-2\sqrt{3/7}/5\pi$
			$p_3p_3 - p_3p_3$ $1/5\pi$	$p_3p_3 - d_3d_3$ $-1/\pi$	$p_3p_3 - d_3d_5$ $-\sqrt{3/7}/5\pi$	$p_3p_3 - d_5d_5$ $-2\sqrt{3/7}/5\pi$
				$d_3d_3 - d_3d_3$ $1/5\pi$	$d_3d_3 - d_3d_5$ $\sqrt{3/7}/5\pi$	$d_3d_3 - d_5d_5$ $2\sqrt{3/7}/5\pi$
					$d_3d_5 - d_3d_5$ $3/35\pi$	$d_3d_5 - d_5d_5$ $6/35\pi$
						$s_1d_3 - d_5d_5$ $12/35\pi$